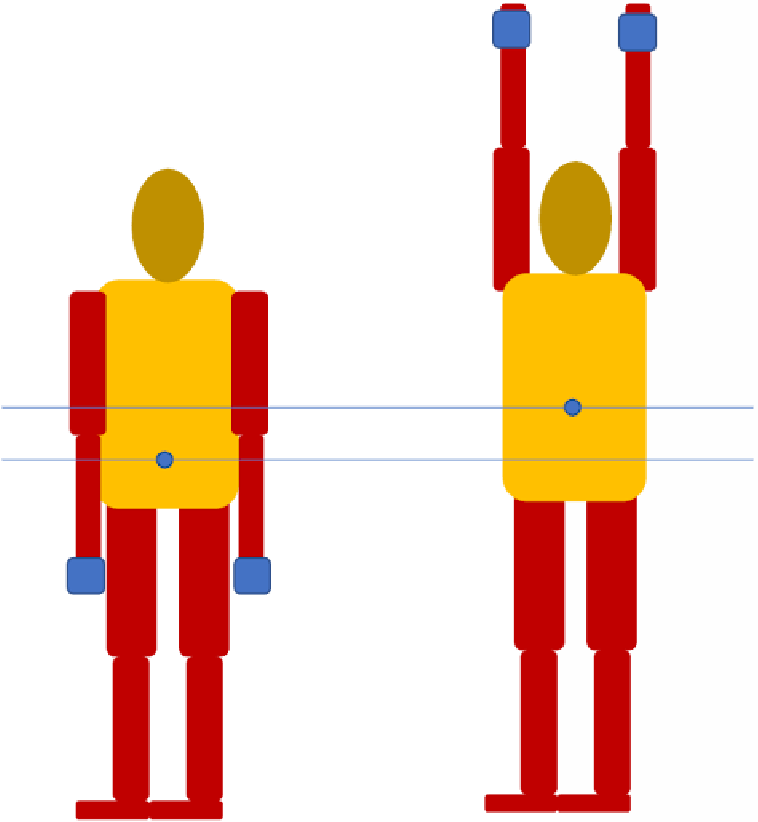

Grade 11S – Physics

Unit Two: Mechanics

Chapter 9: System of Particles

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Think then Solve

Exercise 1

Two particles A and B of respective masses $m_1 = 1.5kg$ & $m_2 = 0.5kg$ are submitted to the action of external forces, thus they move in XOY plane.

The particle A is affected by a force $\vec{F}_1 = 3\vec{i}$ and the particle B is affected by a force $\vec{F}_2 = -\vec{i} + 2\vec{j}$.

The particles A and B are initially at rest at $A_0(2,3)$ and $B_0(4,1)$.

1)Determine the position of the center of mass G_0 of the system (A, B) at t=0s.

2)Calculate the acceleration of the particles A & B.

Exercise 1

3) **Determine at any time t, the velocity and position vectors of A and B.**

4) **Determine the position of the center of mass G of the system (A, B) at any time t. Deduce the value of its acceleration.**

5) **Verify that** $m_1\vec{V}_A + m_2\vec{V}_B = (m_1 + m_2)\vec{V}_G$

Exercise 1

$m_1 = 1.5kg$, $m_2 = 0.5kg$, $\vec{F}_1 = 3\vec{i}$, $\vec{F}_2 = -\vec{i} + 2\vec{j}$, $A_0(2,3)$ & $B_0(4,1)$

1)Determine the position of the center of mass G_0 of the system (A, B) at t=0s.

$$x_{COM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$x_{COM} = \frac{1.5 \times 2 + 0.5 \times 4}{1.5 + 0.5}$$

$$x_{COM} = 2.5m$$

$$y_{COM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

$$y_{COM} = \frac{1.5 \times 3 + 0.5 \times 1}{1.5 + 0.5}$$

$$y_{COM} = 2.5m$$

Exercise 1

$m_1 = 1.5kg$, $m_2 = 0.5kg$, $\vec{F}_1 = 3\vec{i}$, $\vec{F}_2 = -\vec{i} + 2\vec{j}$, $A_0(2,3)$ & $B_0(4,1)$

2) Calculate the acceleration of the particles A & B.

$$\vec{a}_A = \frac{\vec{F}_1}{m_1}$$

$$\vec{a}_A = \frac{3\vec{i}}{1.5}$$

$$\vec{a}_A = 2\vec{i}$$

$$\vec{a}_B = \frac{\vec{F}_2}{m_2}$$

$$\vec{a}_B = \frac{-\vec{i} + 2\vec{j}}{0.5}$$

$$\vec{a}_B = -2\vec{i} + 4\vec{j}$$

Exercise 1

$m_1 = 1.5kg$, $m_2 = 0.5kg$, $\vec{F}_1 = 3\vec{i}$, $\vec{F}_2 = -\vec{i} + 2\vec{j}$, $A_0(2,3)$ & $B_0(4,1)$

3) Determine at any time t, the velocity and position vectors of A and B.

$$\vec{V}_A = \vec{a}_A t + \vec{v}_{0A} \Rightarrow \vec{V}_A = (2\vec{i})t + 0 \Rightarrow \vec{V}_A = 2t\vec{i}$$

$$\vec{r}_A = \frac{1}{2}\vec{a}_A t^2 + \vec{v}_{0A} t + \vec{r}_0 \Rightarrow \vec{r}_A = \frac{1}{2}(2\vec{i})t^2 + (0)t + 2\vec{i} + 3\vec{j}$$

$$\vec{r}_A = (t^2 + 2)\vec{i} + 3\vec{j}$$

Exercise 1

$m_1 = 1.5kg$, $m_2 = 0.5kg$, $\vec{F}_1 = 3\vec{i}$, $\vec{F}_2 = -\vec{i} + 2\vec{j}$, $A_0(2,3)$ & $B_0(4,1)$

3) Determine at any time t, the velocity and position vectors of A and B.

$$\vec{V}_B = \vec{a}_B t + \vec{v}_{0B} \Rightarrow \vec{V}_B = (-2\vec{i} + 4\vec{j})t + 0 \Rightarrow \vec{V}_B = -2t\vec{i} + 4t\vec{j}$$

$$\vec{r}_B = \frac{1}{2}\vec{a}_A t^2 + \vec{v}_{0B} t + \vec{r}_{0B} \Rightarrow \vec{r}_A = \frac{1}{2}(-2\vec{i} + 4\vec{j})t^2 + (0)t + 4\vec{i} + \vec{j}$$

$$\vec{r}_B = \left(-t^2 + 4\right)\vec{i} + (2t^2 + 1)\vec{j}$$

Exercise 1

$$x_G = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x_G = \frac{1.5(t^2 + 2) + 0.5(-t^2 + 4)}{1.5 + 0.5}$$

$$x_G = 0.5t^2 + 2.5$$

$$y_G = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$y_G = \frac{1.5(t^2 + 2) + 0.5(-t^2 + 4)}{1.5 + 0.5}$$

$$x_G = 0.5t^2 + 2.5$$

Exercise 1

4) Determine the position of the center of mass G of the system (A, B) at any time t. Deduce the value of its acceleration

$$\vec{r}_G = X_G\vec{i} + Y_G\vec{j}$$

$$\vec{r}_G = (0.5t^2 + 2.5)\vec{i} + (0.5t^2 + 2.5)\vec{j}$$

$$\vec{V}_G = (\vec{r}_G)' = t\vec{i} + t\vec{j}$$

$$\vec{a}_G = (\vec{V}_G)' = \vec{i} + \vec{j}$$

Exercise 1

5)Verify that $m_1\vec{V}_A + m_2\vec{V}_B = (m_1 + m_2)\vec{V}_G$

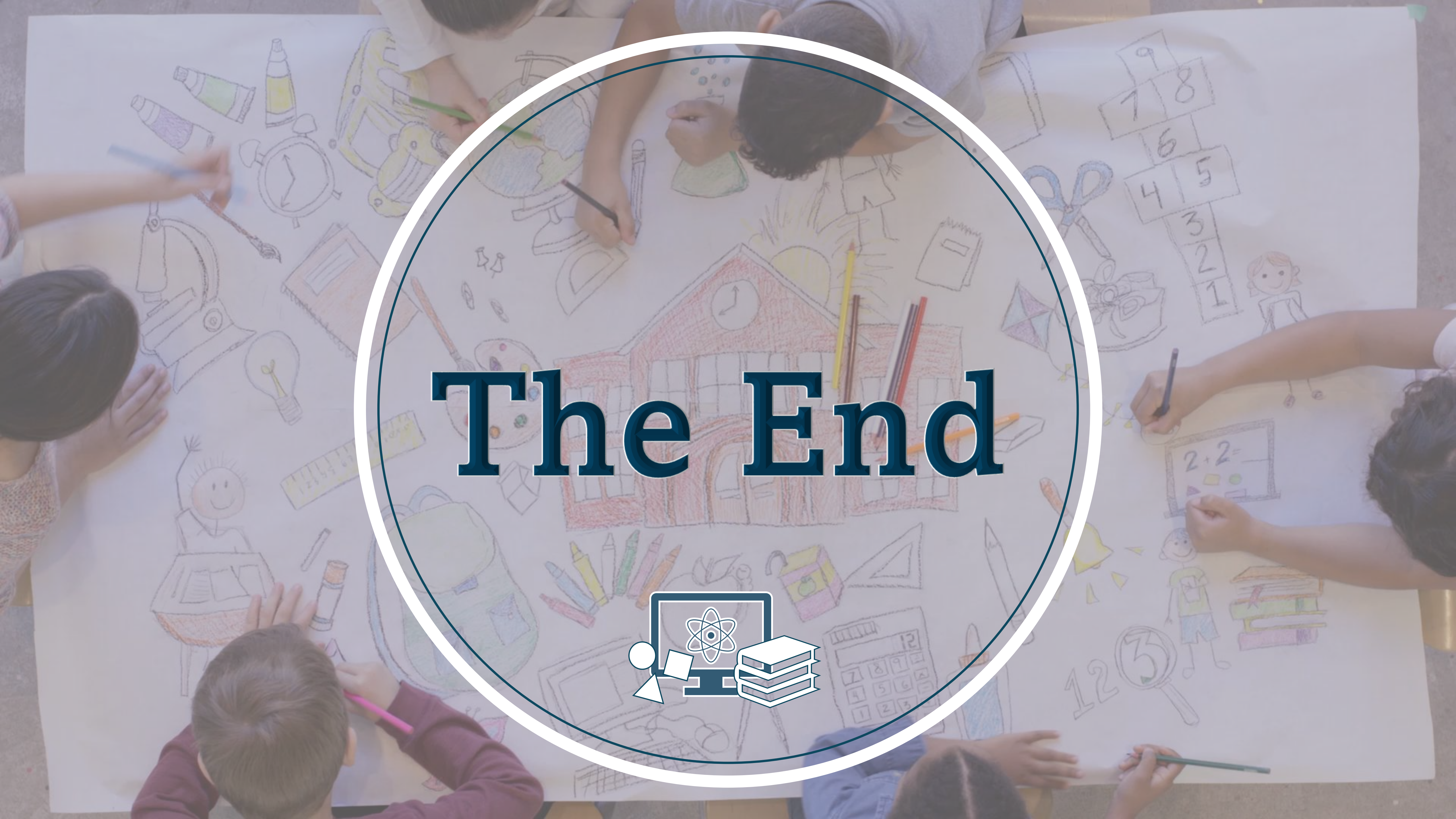
$$m_1\vec{V}_A + m_2\vec{V}_B = 1.5(2t\vec{i}) + 0.5(-2t\vec{i} + 4t\vec{j})$$

$$m_1\vec{V}_A + m_2\vec{V}_B = 3t\vec{i} - t\vec{i} + 2t\vec{j} \Rightarrow m_1\vec{V}_A + m_2\vec{V}_B = 2t\vec{i} + +2\vec{j}$$

$$(m_1 + m_2)\vec{V}_G = (1.5 + 0.5)(t\vec{i} + t\vec{j})$$

$$(m_1 + m_2)\vec{V}_G = 2t\vec{i} + 2t\vec{j}$$

$$m_1\vec{V}_A + m_2\vec{V}_B = (m_1 + m_2)\vec{V}_G$$



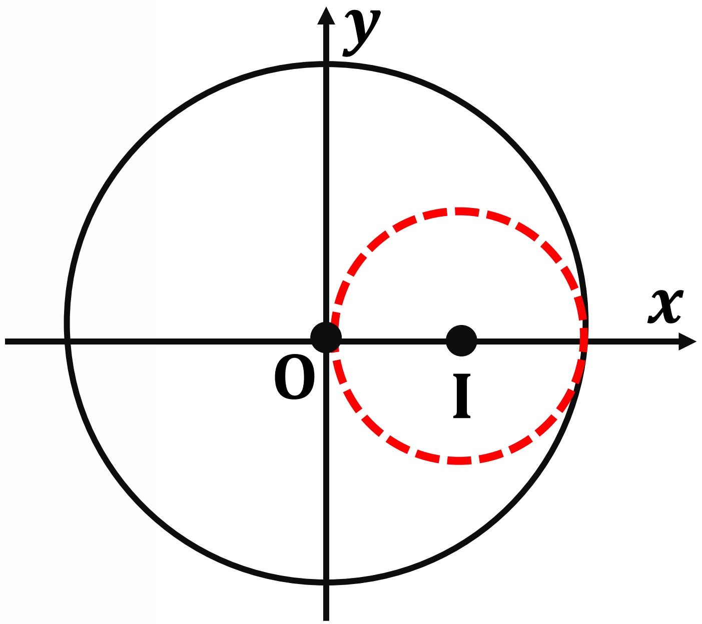
The End

Think then Solve

Exercise 2 Homogenous disc (D)

The adjacent figure shows a homogenous disc (D) of radius R and mass M.

A circular portion is removed out as shown with the dotted line.

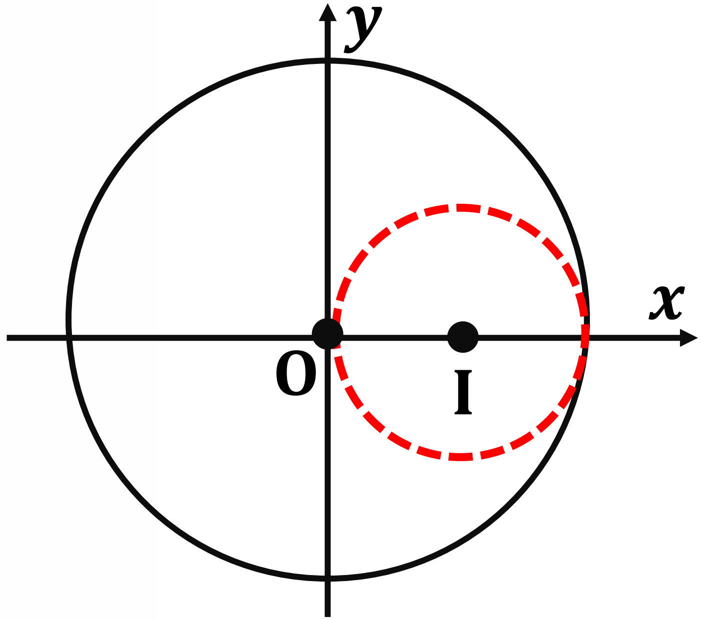


1) Determine the masses of the removed part and the remaining part in terms of M

2) Locate, using the given coordinates system, the center of the mass of the remaining part.

Exercise 2 Homogenous disc (D)

1)Determine the masses of the removed part and the remaining part in terms of M



Since the disc is homogenous then:

$$\frac{m_{disc}}{A_{disc}} = \frac{m_{removed}}{A_{removed}} = \frac{m_{remains}}{A_{remains}}$$

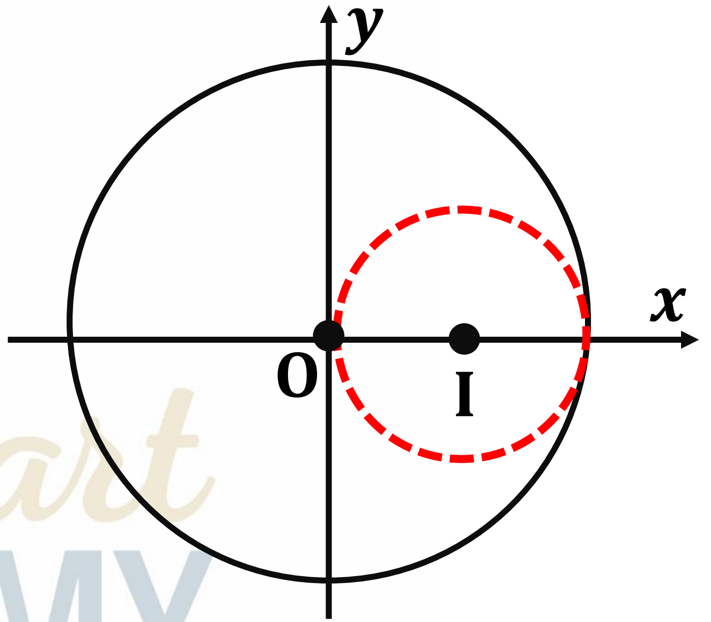
$$\frac{M}{\pi R^2} = \frac{m_{removed}}{\left[\frac{\pi R^2}{4}\right]} = \frac{m_{remains}}{\left[\pi R^2 - \frac{\pi R^2}{4}\right]}$$

Exercise 2 Homogenous disc (D)

$$\frac{M}{\pi R^2} = \frac{m_{removed}}{\left[\frac{\pi R^2}{4}\right]}$$

$$m_{removed} = \frac{M \times \left[\frac{\pi R^2}{4}\right]}{\pi R^2}$$

$$m_{removed} = \frac{M}{4}$$



Exercise 2 Homogenous disc (D)

$$\frac{M}{\pi R^2} = \frac{m_{remains}}{\left[\pi R^2 - \frac{\pi R^2}{4}\right]}$$

$$m_{remains} = \frac{M \times \left[\pi R^2 - \frac{\pi R^2}{4}\right]}{\pi R^2}$$

$$m_{remains} = \frac{M \times \left[\frac{3\pi R^2}{4}\right]}{\pi R^2}$$

$$m_{remains} = \frac{3M}{4}$$

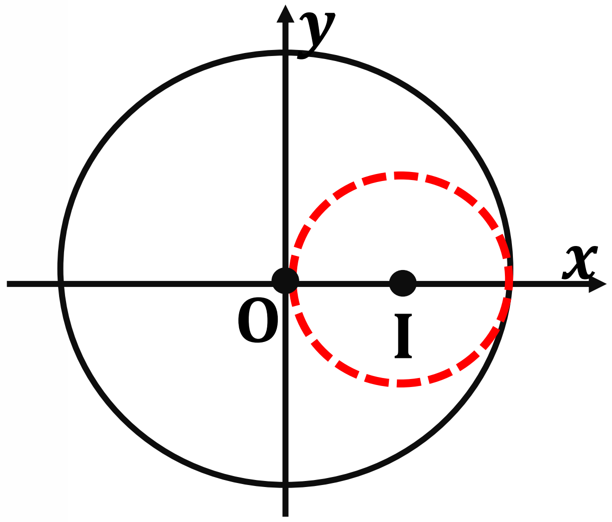
$$m_{remain} = M - m_{removed}$$

$$m_{remain} = M - \frac{M}{4}$$

$$m_{remain} = \frac{3M}{4}$$

Exercise 2 Homogenous disc (D)

2) Locate, using the given coordinates system, the center of the mass of the remaining part.



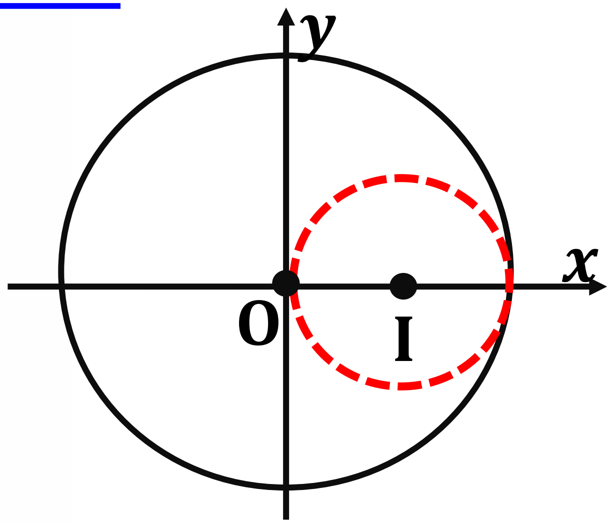
The whole disc can be considered as a system made up of two parts:

The removed part of mass $m_{removed} = \frac{M}{4}$ and center I($\frac{R}{2}$, 0).

The remaining part of mass $m_{remain} = \frac{3M}{4}$ and center A(x, y).

Exercise 2 Homogenous disc (D)

$$x_0 = \frac{m_{removed} \times x_{removed} + m_{remain} \times x_{remain}}{m_{removed} + m_{remain}}$$



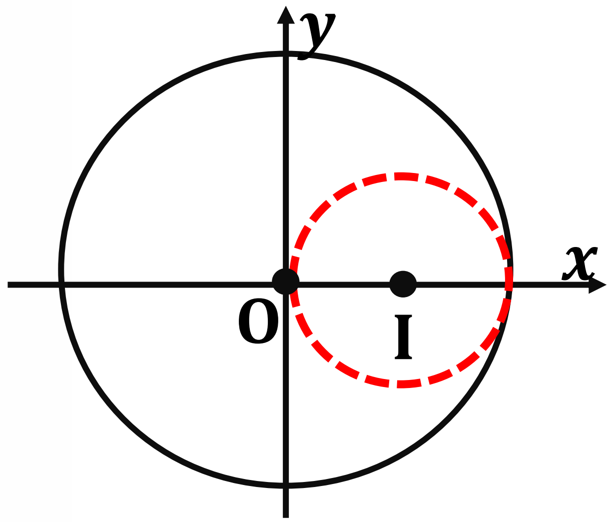
$$x_0 = \frac{\frac{M}{4}\left[\frac{R}{2}\right] + \frac{3M}{4}.x}{M}$$

$$0 = \frac{\frac{M.R}{8} + \frac{3M}{4}.x}{M}$$

$$x = -\frac{R}{6}$$

Exercise 2 Homogenous disc (D)

$$y_0 = \frac{m_{removed} \times y_{removed} + m_{remain} \times y_{remain}}{m_{removed} + m_{remain}}$$

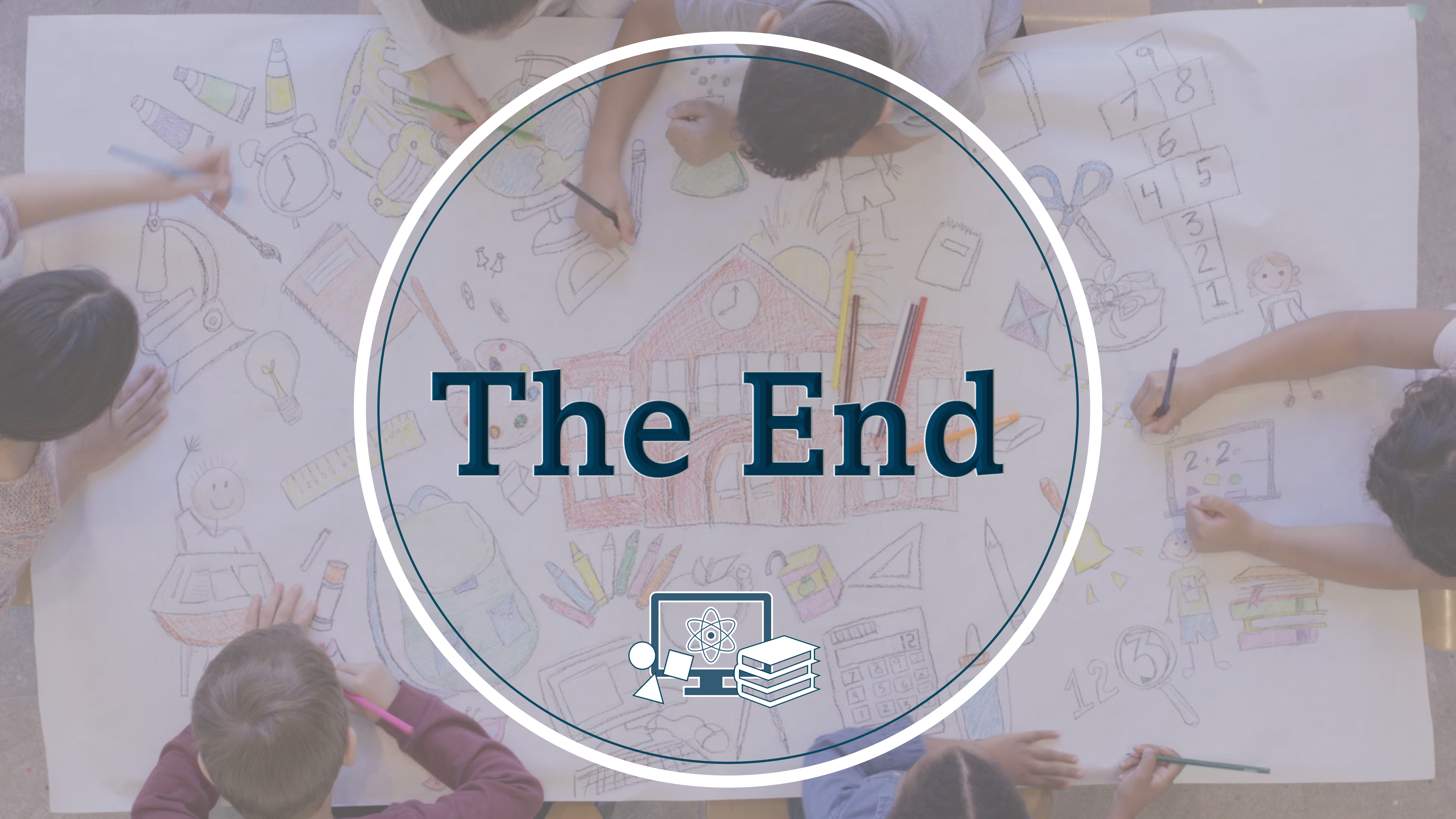


$$0 = \frac{\frac{M}{4}(0) + \frac{3M}{4}.y}{M}$$

$$0 = \frac{0 + \frac{3M}{4}.y}{M}$$

$$y = 0$$

Then the center of the remaining part is located at A($-\frac{R}{6}$, 0)



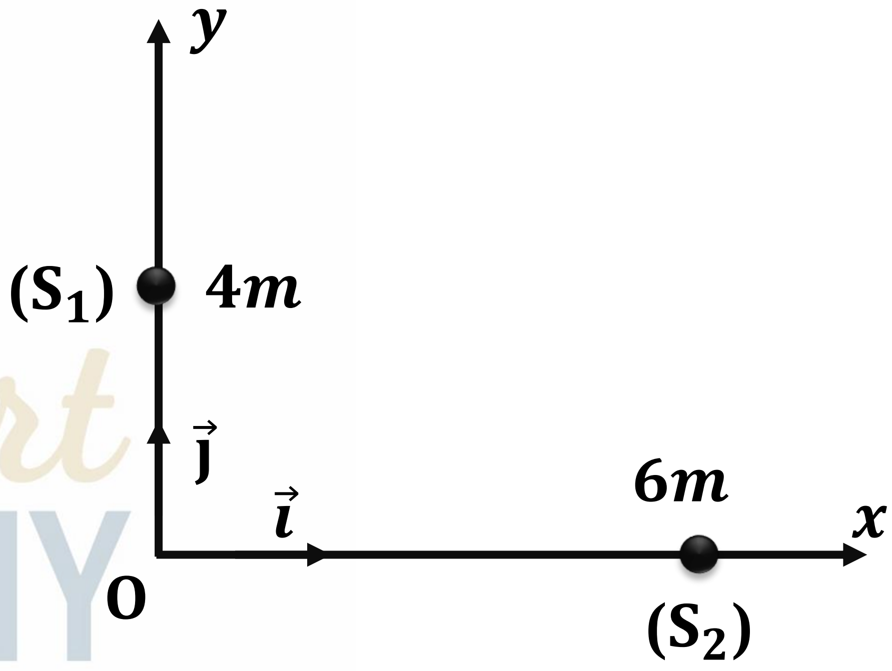
The End

Think then Solve

Exercise 3

Two solids (S_1) and (S_2) taken as particles of respective masses $m_1 = 2kg$ and $m_2 = 4kg$, are placed in the space frame of reference $(O, \vec{i}, \vec{j})$ as shown in the adjacent figure

Part A: (S_1) and (S_2) are at rest:



1) **Determine X_G and Y_G the coordinate of the center of mass [(S_1);(S_2)].**
2) **Deduce the position vector of the center of mass of the system [(S_1);(S_2)].**

Exercise 3

Part A: (S_1) and (S_2) are at rest:

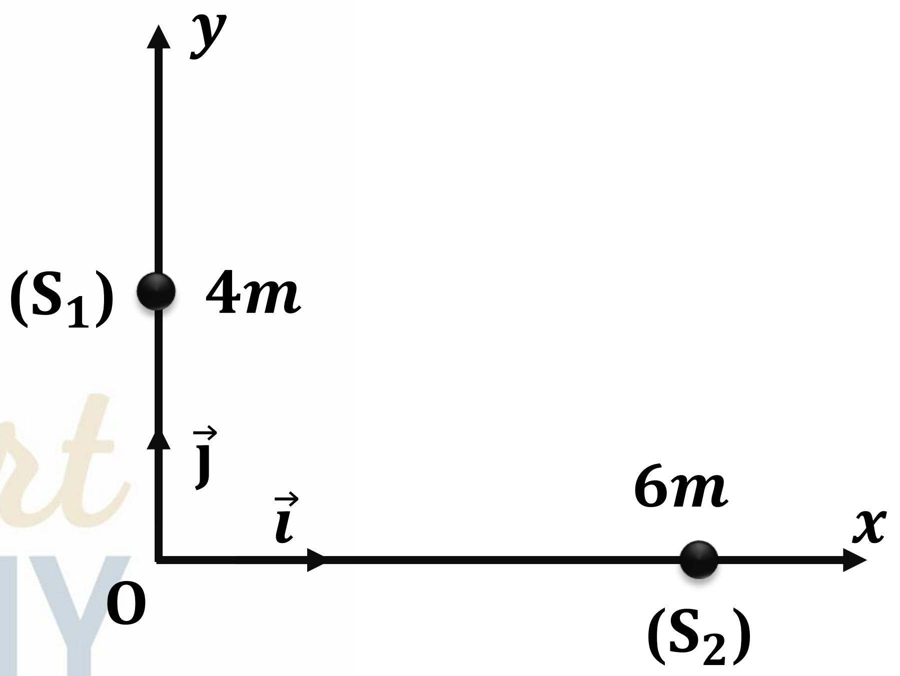
1) Determine X_G and Y_G the coordinate of the center of mass $[(S_1);(S_2)]$.

$$X_G = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$X_G = \frac{2(0) + 4(6)}{2+4} \Rightarrow X_G = 4m$$

$$Y_G = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$Y_G = \frac{2(4) + 4(0)}{2+4} \Rightarrow Y_G = \frac{8}{6}m$$

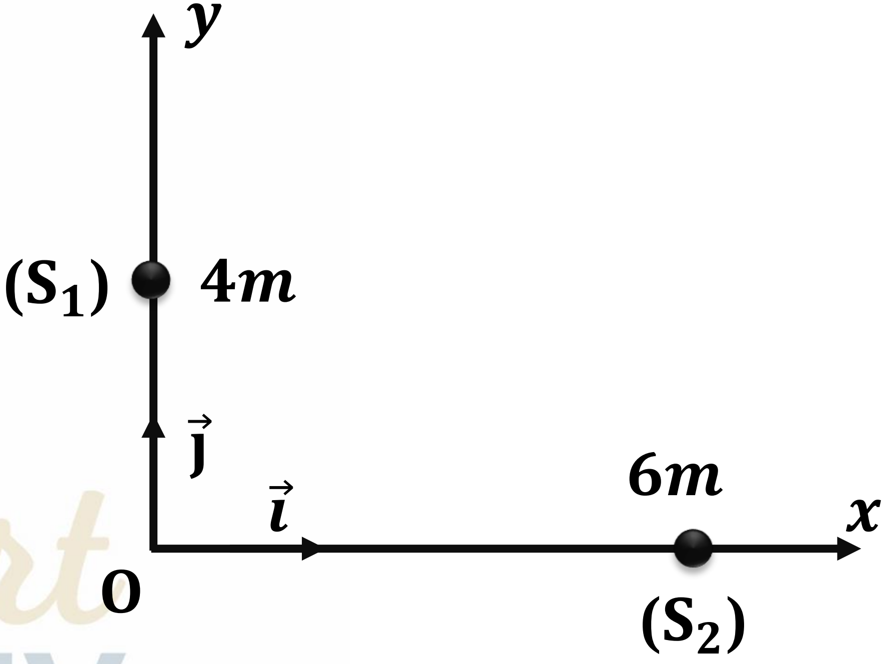


Exercise 3

2) Deduce the position vector of the center of mass of the system $[(S_1);(S_2)]$.

$$\vec{r}_G = X_G\vec{i} + Y_G\vec{j}$$

$$\vec{r}_G = 4\vec{i} + \frac{8}{6}\vec{j}$$



Exercise 3

Part B: Motion of the center of mass:

At instant $t_0 = 0s$ (S_1) is submitted to the force $\vec{F}_1 = a.\vec{i}$ where a is constant while (S_2) is submitted to the external force $\vec{F}_2 = 8.\vec{j}$.

1) Calculate the acceleration vector $\vec{a}_1$ (in terms of a) and $\vec{a}_2$ of (S_1) and (S_2) respectively at any time t.
2) Deduce $\vec{r}_1$ (in terms of a) and $\vec{r}_2$ the position vectors of (S_1) and (S_2) respectively at any time t.
3) Show that a=6N knowing that (S_1) meets (S_2) at the instant t=2sec.
4) Determine the position of the center of mass of the system [(S_1);(S_2)] and deduce its acceleration.
5) Verify the theorem of center of mass $M\vec{a}_G = m_1\vec{a}_1 + m_2\vec{a}_2$, where $M = m_1 + m_2$

Exercise 3

At $t_0 = 0s$; $\vec{F}_1 = a.\vec{i}$ & $\vec{F}_2 = 8.\vec{j}$.

1) Calculate the acceleration vector $\vec{a}_1$ (in terms of a) and $\vec{a}_2$ of (S_1) and (S_2) respectively at any time t.

$$\vec{F}_1 = m_1\vec{a}_1 \Rightarrow a.\vec{i} = 2\vec{a}_1$$

$$\vec{a}_1 = \frac{a.\vec{i}}{2}$$

$$\vec{F}_2 = m_2\vec{a}_2 \Rightarrow 8.\vec{j} = 4\vec{a}_2 \Rightarrow \vec{a}_2 = \frac{8.\vec{j}}{4}$$

$$\vec{a}_2 = 2.\vec{j}$$

Exercise 3

2) Deduce $\vec{r}_1$ (in terms of a) and $\vec{r}_2$ the position vectors of (S_1) and (S_2) respectively at any time t.

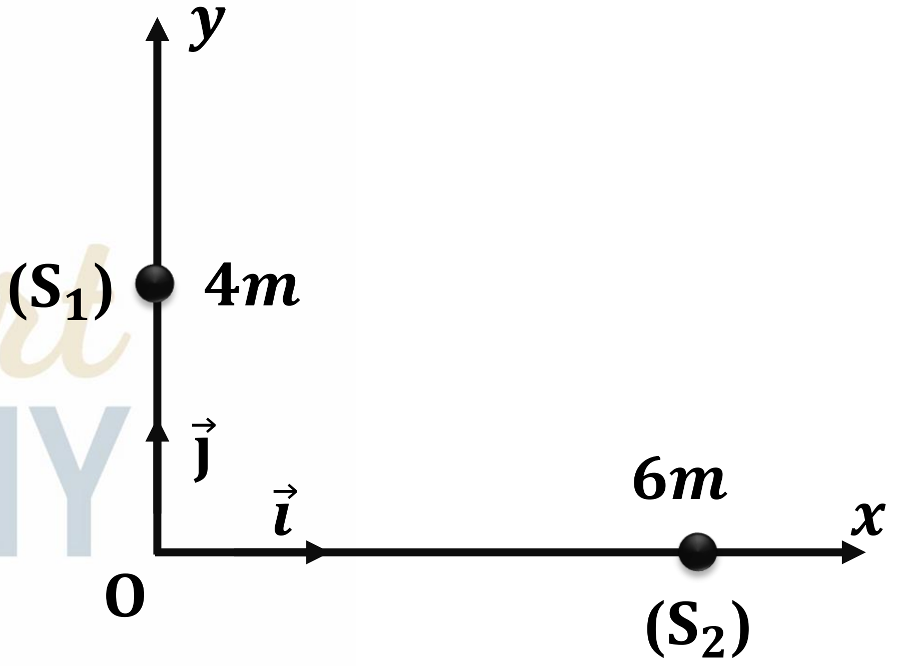
$$\vec{r}_1 = \frac{1}{2}\vec{a}_1 t^2 + \vec{v}_0 t + \vec{r}_0 \quad \Rightarrow \quad \vec{r}_1 = \frac{1}{2}\left(\frac{a}{2}.\vec{i}\right)t^2 + (0)t + 4\vec{j}$$

$$\vec{r}_1 = \frac{a}{4}t^2\vec{i} + 4\vec{j}$$

$$\vec{r}_2 = \frac{1}{2}\vec{a}_2 t^2 + \vec{v}_0 t + \vec{r}_0$$

$$\vec{r}_2 = \frac{1}{2}(2.\vec{j})t^2 + (0)t + 6\vec{i}$$

$$\vec{r}_2 = 6\vec{i} + t^2\vec{j}$$



Exercise 3

3) Show that a=6N knowing that (S_1) meets (S_2) at the instant t=2sec.

(S_1) meets (S_2) at t=2sec then $\vec{r}_1 = \vec{r}_2$

$$\frac{a}{4}t^2\vec{i} + 4\vec{j} = 6\vec{i} + t^2\vec{j} \Rightarrow \frac{at^2}{4} = 6 \Rightarrow at^2 = 24$$

For t=2s: $\Rightarrow a(2)^2 = 24 \Rightarrow 4a = 24 \Rightarrow a = \frac{24}{4}$

$$a = 6$$

$$\vec{r}_1 = \frac{a}{4}t^2\vec{i} + 4\vec{j} \Rightarrow \vec{r}_1 = \frac{6}{4}t^2\vec{i} + 4\vec{j} \Rightarrow \vec{r}_1 = 1.5t^2\vec{i} + 4\vec{j}$$

Exercise 3

4) **Determine the position of the center of mass of the system $[(S_1);(S_2)]$ and deduce its acceleration.**

$$\vec{r}_G = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} \Rightarrow \vec{r}_G = \frac{2(1.5t^2\vec{i} + 4\vec{j}) + 4(6\vec{i} + t^2\vec{j})}{2+4}$$

$$\vec{r}_G = \frac{3t^2\vec{i} + 8\vec{j} + 24\vec{i} + 4t^2\vec{j}}{2+4} \Rightarrow \vec{r}_G = \frac{(3t^2+24)\vec{i} + (4t^2+8)\vec{j}}{6}$$

$$\vec{r}_G = (\frac{1}{2}t^2 + 4)\vec{i} + (\frac{2}{3}t^2 + \frac{4}{3})\vec{j}$$

Exercise 3

$$\vec{r}_G = (\frac{1}{2}t^2 + 4)\vec{i} + (\frac{2}{3}t^2 + \frac{4}{3})\vec{j}$$

The velocity vector of the CM is the derivative of the position vector of CM

$$\vec{V}_G = \vec{r}'_G = t.\vec{i} + \frac{4}{3}t.\vec{j}$$

The acceleration vector of the CM is the derivative of the position vector of CM

$$\vec{a}_G = \vec{V}'_G = \vec{i} + \frac{4}{3}\vec{j}$$

Exercise 3

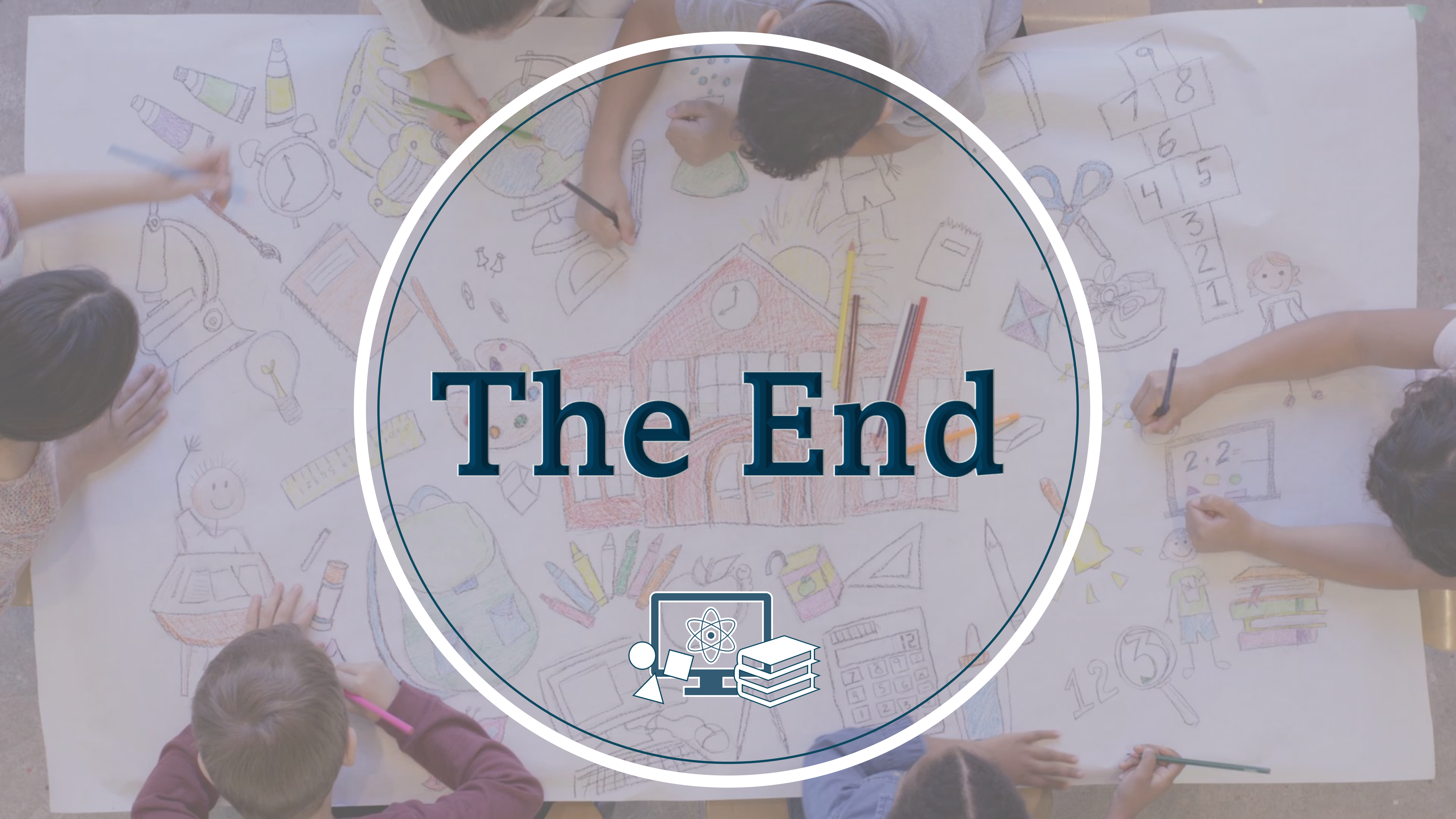
5) Verify the theorem of center of mass $M\vec{a}_G = m_1\vec{a}_1 + m_2\vec{a}_2$, where $M = m_1 + m_2$

$$m_1\vec{a}_1 + m_2\vec{a}_2 = 2(3\vec{i}) + 4(2\vec{j}) \Rightarrow m_1\vec{a}_1 + m_2\vec{a}_2 = 6\vec{i} + 8\vec{j}$$

$$M\vec{a}_G = (2+4)\left[\vec{i} + \frac{4}{3}\vec{j}\right]$$

$$M\vec{a}_G = 6\vec{i} + 8\vec{j}$$

The theorem of center of mass $M\vec{a}_G = m_1\vec{a}_1 + m_2\vec{a}_2$ is verified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